IOSR Journal of Engineering (IOSR JEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
PP 13-17

Survival of Biological Species under The Effect of Toxins: A Study Through Mathematical Models

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Abstract: *In this paper, we have tried to study the effect of toxins on logistically growing biological species. We have proposed some non linear mathematical models to study the effect in various cases, such as:*
(i) The toxin is being emitted in to the environment by some external source.
(ii) The toxin is being discharged by the species itself.
(iii) Simultaneous effect of two toxins.
These models involve system of non linear ordinary differential equations. The qualitative analysis has been done using stability theory. These models have been simulated using MATLAB.
Keywords: *Biological species, Toxins, Mathematical models, Simulation.*

I. Introduction

In past several decades, there are many studies (Agarwal, *et al.*2016; Kumar *et al.*, 2016; Misra*et al.*, 2015; Agarwal, *et al.*2015; Kumar *et al.*, 2015;Khare et al. 2011;Dubey et al., 2010;Agrawal,*et al.*, 2000;Agrawal, 1999; Shukla *et al.*, 1999; Shukla andDubey 1996; Deluna and Hallam, 1987) present to study the effect of toxicants on biological species. Some of these studies (Agrawal,*et al.*, 2000;Agrawal, 1999; Shukla *et al.*, 1999; Shukla andDubey 1996) show the growth of biological species in toxic environment. These studies are based on mathematical models and their analyses.

In case first, Agrawal, 1999 has studied the behaviour of logistically growing biological species which is living in a toxic environment. This toxic environment contains a single toxicant and at a constant rate this toxicant is being emitted in to the environment by some external source. Constant emission of this toxicant increases the concentration level.

The case second is similar to the case first with different situation in which affected species emits the toxicant itself in to the environment.

In case third,Agrawal, 1999has studied the simultaneous effect of two toxicants on a logistically growing biological species. In this case, both toxicants being emitted in to the environment by some external sources.

In all of these studies, a mathematical model based on system of non-linear differential equations has been proposed and analyzed in each case. These studies play a great role to study the growth of the biological species in toxic environment. These studies will be more realistic after including simulation graphs in these studies. So, in this paper, we give a short description of mathematical models and simulation graphs of the proposed cases.

II. Mathematical Models And Their Simulation Graphs

Case (i): The toxin is being emitted in to the environment by some external source

In this case, we have postulated that a single toxicant is being constantlyemitted in to the environment with a constant rate Q. $N(t)$is the density of the biological population at time t, which is logistically growing and surviving in this toxic environment. The environmental concentration of this toxicant is $T(t)$ at time t. We assume that $U(t)$ is the concentration of toxicant $T(t)$, taken up by the biological species $N(t)$ at time t. With these assumptions, the model is:

$$\frac{dN}{dt} = \left[r(U) - \frac{r_0 N}{K(T)}\right] N$$
$$\frac{dT}{dt} = Q - \delta_0 T - \alpha NT + \pi\nu NU \qquad (1)$$
$$\frac{dU}{dt} = -\delta_1 U + \alpha NT - \nu NU$$

The model (1) has two equilibrium points $E_1\left(0, \frac{Q}{\delta_0}, 0\right)$and $E_2(\tilde{N}, \tilde{T}, \tilde{U})$, where $\tilde{N}$, $\tilde{T}$ and $\tilde{U}$ are the positive solutions of system of equations

$$N = \frac{r(U)K(T)}{r_0}; \qquad T = \frac{Q(\delta_1 + \nu N)}{f(N)}; \qquad U = \frac{Q\alpha N}{f(N)}$$

and $f(N) = \delta_0\delta_1 + (\delta_0\nu + \alpha\delta_1)N + \alpha\nu(1-\pi)N^2$

The stability analysis (Agrawal, 1999) of the model (1) shows that the equilibrium point $E_1\left(0, \frac{Q}{\delta_0}, 0\right)$ is a saddle point which is stable in $T - U$ directions and unstable in N direction. The equilibrium point $E_2(\tilde{N}, \tilde{T}, \tilde{U})$ is locally and globally stable under certain conditions.

Simulation Graphs: In Figure 1, we draw the simulation graph of the model (1). This simulation graph describe the continuous growth of biological species with respect to the time for different values of emission rate of external toxicant Q. Here, we assume the defined functions $r(U)$ and $K(T)$ in model such as:

$$r(U) = r_0 - r_1 U \qquad \text{and} \qquad K(T) = K_0 - \frac{b_1 T}{1 + b_2 T}$$

and the value of parameters are assumed as:

$$r_0 = 0.20, \quad r_1 = 0.80, \quad K_0 = 10.0, \quad b_1 = 0.010, \quad b_2 = 1.0,$$
$$\delta_0 = 0.040, \quad \pi = 0.80, \quad \nu = 0.040, \quad \delta_1 = 0.00050, \quad \alpha = 0.0010.$$

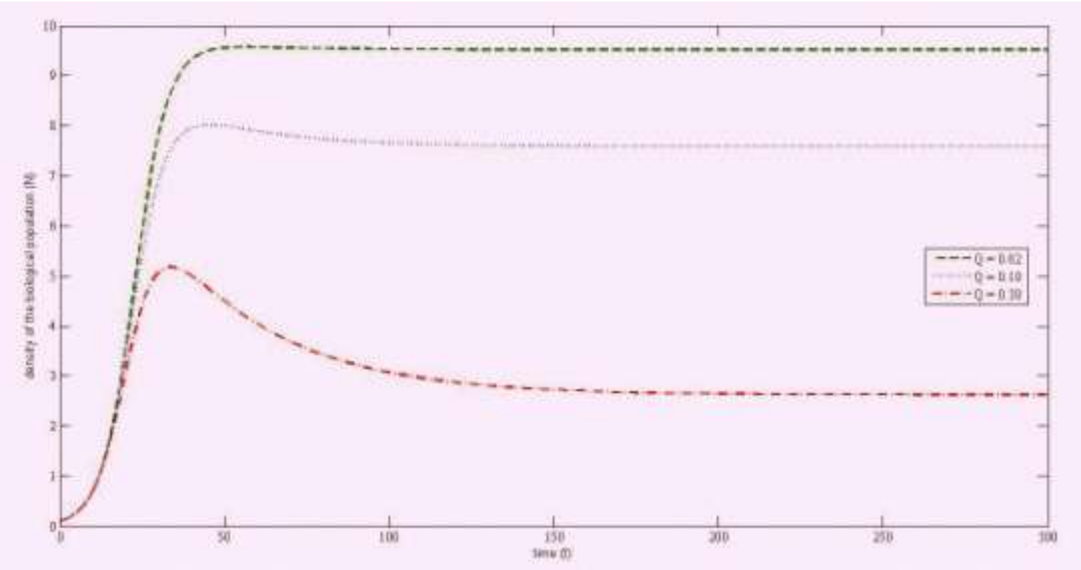

Figure 1: *Growth of density of biological population* ($\boldsymbol{N}$) *with respect to time* ($\boldsymbol{t}$).

Case (ii): The toxin is being discharged by the species itself

This case is similar to the first case with a different situation in which toxicant emitted in the environment by the biological species itself. This model is relevant to the case of human population. For this situation, the model is given as:

$$\frac{dN}{dt} = \left[r(U) - \frac{r_0 N}{K(T)}\right]N$$
$$\frac{dT}{dt} = \lambda N - \delta_0 T - \alpha NT + \pi\nu NU \qquad (2)$$
$$\frac{dU}{dt} = -\delta_1 U + \alpha NT - \nu NU$$

The model (2) has two equilibrium points $E_3(0,0,0)$ and $E_4(N^*, T^*, U^*)$, where N^*, T^* and U^* are the positive solutions of system of equations

$$N = \frac{r(U)K(T)}{r_0}; \qquad T = \frac{\lambda N(\delta_1 + \nu N)}{f(N)}; \qquad U = \frac{\lambda\alpha N^2}{f(N)}$$

and $f(N) = \delta_0\delta_1 + (\delta_0\nu + \alpha\delta_1)N + \alpha\nu(1-\pi)N^2$

The stability analysis (Agrawal, 1999) of the model (2) shows that the equilibrium point $E_3(0,0,0)$ is a saddle point which is stable in $T - U$ directions and unstable in N direction. The equilibrium point $E_4(N^*, T^*, U^*)$ is locally and globally stable in under certain conditions.

Simulation Graphs: Similarly, as case (i), we assume the defined functions $r(U)$ and $K(T)$ in model (2) such as:

$$r(U) = r_0 - r_1 U \qquad \text{and} \qquad K(T) = K_0 - \frac{b_1 T}{1 + b_2 T}$$

and the value of parameters are assumed as:

$$r_0 = 0.20, \quad r_1 = 0.80, \quad K_0 = 10.0, \quad b_1 = 0.010, \quad b_2 = 1.0,$$
$$\delta_0 = 0.040, \quad \pi = 0.80, \quad \nu = 0.040, \quad \delta_1 = 0.00050, \quad \alpha = 0.0010.$$

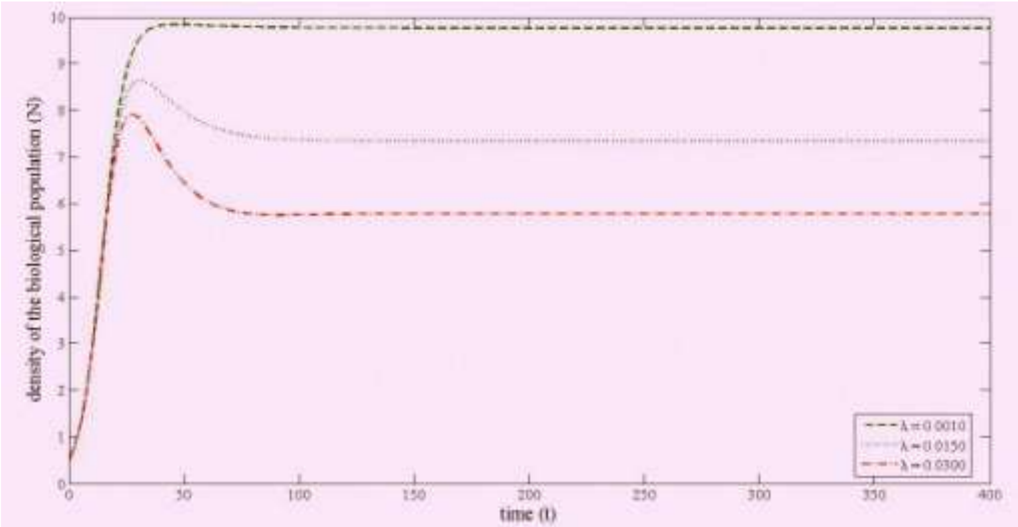

Figure 2: *Growth of density of biological population* (N) *with respect to time* (t).

Figure 2 shows the growth of density of biological population with respect to the time at different emission rates of toxicant which is emitted in the environment by the biological population itself. This graph shows that as the emission rate of toxicant increases the level of biological population density decreases.

Case (iii): Simultaneous effect of two toxins

Here, we consider a logistically growing biological population with density $N(t)$ in the environment and simultaneously affected by two different types of toxicants with environment concentrations $T_1(t)$ and $T_2(t)$ (both toxicants are constantly emitted in the environment at the rates Q_1 and Q_2 respectively, from some external sources). These toxicants are correspondingly uptaken by the biological population at different concentration rates $U_1(t)$and $U_2(t)$. These toxicants decrease the growth rate of biological population. Keeping these views in mind, the model is:

$$\frac{dN}{dt} = \left[r(U_1, U_2) - \frac{r_0 N}{K(T_1, T_2)}\right] N$$
$$\frac{dT_1}{dt} = Q - \delta_1 T_1 - \gamma_1 T_1 N + \pi_1 \nu_1 N U_1$$
$$\frac{dT_2}{dt} = \lambda N - \delta_2 T_2 - \gamma_2 T_2 N + \pi_2 \nu_2 N U_2 (3)$$
$$\frac{dU_1}{dt} = \gamma_1 T_1 N - \beta_1 U_1 - \nu_1 N U_1$$
$$\frac{dU_2}{dt} = \gamma_2 T_2 N - \beta_2 U_2 - \nu_2 N U_2$$

The mathematical model (3)has two equilibrium points $E_5\left(0, \frac{Q}{\delta}, 0,0,0\right)$and $E_6(\overline{N}, \overline{T_1}, \overline{T_2}, \overline{U_1}, \overline{U_2})$, where $\overline{N}, \overline{T_1}, \overline{T_2}, \overline{U_1}$ and $\overline{U_2}$ are the positive solutions of system of equations

$$N = \frac{r(U_1, U_2) K(T_1, T_2)}{r_0}; \qquad T_1 = \frac{Q(\beta_1 + \nu_1 N)}{f_1(N)}; \qquad T_1 = \frac{\lambda N(\beta_2 + \nu_2 N)}{f_2(N)},$$
$$U_1 = \frac{Q \gamma_1 N}{f_1(N)}, \qquad U_2 = \frac{\lambda \gamma_2 N^2}{f_2(N)}$$

where, $f_1(N) = \delta_1 \beta_1 + (\gamma_1 \beta_1 + \delta_1 \nu_1) N + \gamma_1 \nu_1 (1 - \pi_1) N^2$
$f_2(N) = \delta_2 \beta_2 + (\gamma_2 \beta_2 + \delta_2 \nu_2) N + \gamma_2 \nu_2 (1 - \pi_2) N^2$

The stability analysis (Agrawal, 1999) of the mathematical model (3) shows that the equilibrium point $E_5\left(0, \frac{Q}{\delta}, 0,0,0\right)$ is a saddle point which is stable in $T_1 - T_2 - U_1 - U_2$ directions and unstable in N direction. The equilibrium point $E_6(\overline{N}, \overline{T_1}, \overline{T_2}, \overline{U_1}, \overline{U_2})$, is locally and globally stable under certain conditions.

Simulation Graphs: In the mathematical model (3), we assume the defined functions $r(U_1, U_2)$ and $K(T_1, T_2)$ such as:

$$r(U_1, U_2) = r_0 - r_1 U_1 - r_2 U_2 \text{and} \qquad K(T_1, T_2) = K_0 - \frac{b_{11} T_1}{1 + b_{12} T_1} - \frac{b_{21} T_2}{1 + b_{22} T_2}$$

and the value of parameters are assumed as:

$$K_0 = 10.0, \quad b_{11} = 0.02, \quad b_{12} = 1.0, \quad b_{21} = 0.01, \quad b_{22} = 2.0, \quad r_0 = 0.04,$$
$$r_1 = 0.0009, \quad r_2 = 0.0006, \quad \delta_1 = 0.04, \quad \delta_2 = 0.005, \quad \gamma_1 = 0.0004$$
$$\gamma_2 = 0.0008, \quad \pi_1 = 0.0004, \quad \pi_2 = 0.0006, \quad \nu_1 = 0.0005,$$
$$\nu_2 = 0.0001, \quad \beta_1 = 0.06, \quad \beta_2 = 0.08.$$

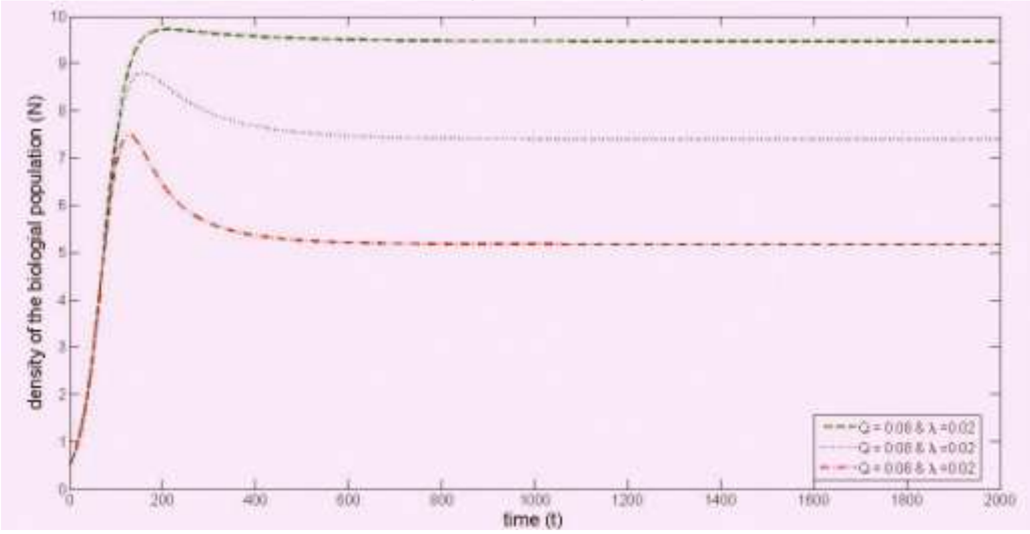

Figure 3: *Growth of density of biological population* (N) *with respect to time* (t).

Figure 3 shows the growth of density of biological population with respect to the time at different emission rates of toxicant which is emitted in the environment by the biological population itself. This graph shows that as the emission rate of toxicantincreases the level of biological populationdensity decreases.

III. Conclusion

In this paper, we give a short description including simulation graphs of the studies based on effect of toxins on logistic growing biological species on the following cases:

(i) The toxin is being emitted in to the environment by some external source.
(ii) The toxin is being discharged by the species itself.
(iii) Simultaneous effect of two toxins.

After including these simulation graphs these studies become more realistic.

References

[1]. Agrawal, A.K. (1999): Effects of toxicants on biological species: some non-linear mathematical models and their analyses. Ph.D. thesis, Department of Mathematics, I.I.T. Kanpur, U.P., INDIA.

[2]. Agarwal, A.K, Khan, A.W. and Agrawal, A.K. (2016): The effect of an external toxicant on a biological species in case of deformity: A model, *Modeling Earth System and Environment*,**2(3)**, 1-8.

[3]. Agrawal, A.K., Sinha, P., Dubey, B. and Shukla, J.B. (2000): Effects of two or more toxicants on a biological species: a non linear mathematical model and its analysis, *Dwivedi AP (ed) Mathematical analysis and applications, Narosa Publishing House, New Delhi*, 97–113.

[4]. Agarwal, A.K., Khan, A.W. and Agrawal A.K. (2015): A model for the adverse effect of two toxicants causing deformity in a subclass of a biological species, *International Journal of Pure and Applied Mathematics*, **110(3)**, 447-462.

[5]. Deluna JT, Hallam TG (1987): Effect of toxicants on population: aqualitative approach IV. Resource–consumer– toxicant models.*Ecol Modell*,*35*, 249–273.

[6]. Dubey, B., Shukla, J.B., Sharma, S., Agrawal, A.K., and Sinha, P. (2010). A mathematical model for chemical defense mechanism of two competing species, *NonlinearAnalysis: Real World Application*, **11(2)**, 1143–1158.

[7]. Khare, S., Misra, O.P., Singh, C., andDhar, J. (2011). Role of delay on Planktonic ecosystem in the presence of a toxic producing phtoplankton, *International Journal of Differential Equations*, 1-16.

[8]. Kumar, A., Agrawal, A.K., Hasan, A. and Misra, A.K. (2016): Modeling the Effect of Toxicant on the Deformity in a Subclass of a Biological Species, *Modeling Earth System and Environment*, **2(1)**, 1-14.

[9]. Kumar, A, Khan, A.W. and Agrawal, A.K. (2015): Modeling the Simultaneous Effect of Two Toxicants Causing Deformity in a Subclass of Biological Population, *IOSR-JM*, **11(6)**, 70-82.

[10]. Misra, A.K., Verma, M., &Venturino, E. (2015). Modeling the control of atmospheric carbon dioxide through reforestation: effect of time delay, *Modeling Earth Systems and Environment*, **1(3)**, 1-17.

[11]. Shukla, J.B. and Agrawal, A.K. (1999): Some mathematical models in ecotoxicology: Effects of toxicants on biological species, *Sadhana*, **24(1-2)**, 25–40.

[12]. Shukla, J.B. and Dubey, B. (1996): Simultaneous effects of two toxicants on biological species: A mathematical model, *J. Biol. Systems* **4**, 109-130.